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NOTE ON THE NORM ESTIMATES FOR THE WEIGHTED DIFFERENTIATION COMPOSITION OPERATOR BETWEEN SOME SPACES OF ANALYTIC FUNCTIONS ON THE UNIT DISK

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Abstract. We establish an estimation of an upper bound for the norm of a specific linear operator of considerable recent interest that combines three important fundamental linear operators, namely the differentiation operator, the multiplication operator, and the composition operator (collectively, the so-called *weighted differentiation composition operator*). This operator acts between two spaces of analytic functions on the unit disk in the complex plane. In a more general setting, we impose a condition on the linear operator ensuring that the aforementioned estimate is bounded by a specific quantity, which, in a way, generalizes the concept of operator norm in a certain sense. As a corollary of these two estimate type results, we calculate the exact norm of the linear operator between these spaces, in the case where they both are normed linear vector spaces of analytic functions. We also present some noteworthy remarks of independent interest, for the case where one of the spaces is not a normed linear vector space of analytic functions, but rather a Fréchet space.

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