SZEGÖ LIMIT THEOREMS FOR OPERATORS
WITH ALMOST PERIODIC DIAGONALS

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Abstract. The classical Szegö theorems study the asymptotic behaviour of the determinants of the finite sections $P_n T(a) P_n$ of Toeplitz operators, i.e., of operators which have constant entries along each diagonal. We generalize these results to operators which have almost periodic functions on their diagonals.

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