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LOCATION OF THE SPECTRUM OF OPERATOR MATRICES WHICH ARE ASSOCIATED TO SECOND ORDER EQUATIONS

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Abstract. In this paper, second order equations of the form $\ddot{z}(t) + A_0 z(t) + D\dot{z}(t) = 0$ are studied, where A_0 is a uniformly positive operator and $A_0^{-1/2} D A_0^{-1/2}$ is a bounded non-negative operator in a Hilbert space H. This equation is equivalent to the standard first-order equation $\dot{x}(t) = \mathcal{A}x(t)$, where $\mathcal{A}$ has the domain

$$\mathscr{D}(\mathcal{A}) = \left\{ \begin{bmatrix} z \\ w \end{bmatrix} \in \mathscr{D}(A_0^{1/2}) \times \mathscr{D}(A_0^{1/2}) \mid A_0 z + Dw \in H \right\}$$

and is given by

$$\mathcal{A} = \begin{bmatrix} 0 & I \\ -A_0 & -D \end{bmatrix}.$$

The location of the spectrum and the essential spectrum of the semigroup generator $\mathcal{A}$ is described under various conditions on the damping operator D. By means of an example it is shown that in general the spectrum can be quite arbitrary in the closed left half plane.

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