

ORDER REDUCTION OF DISCRETE-TIME ALGEBRAIC RICCATI EQUATIONS WITH SINGULAR CLOSED LOOP MATRIX

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Abstract. We study the general discrete-time algebraic Riccati equation and deal with the case where the closed loop matrix corresponding to an arbitrary solution is singular. In this case the extended symplectic pencil associated with the DARE has 0 as a characteristic root and the corresponding spectral deflating subspace gives rise to a subspace where all solutions of the DARE coincide. This allows for a reduction of the original DARE to an equation of smaller size.

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