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MEROMORPHIC SOLUTIONS OF LINEAR DIFFERENTIAL SYSTEMS, PAINLEVÉ TYPE FUNCTIONS

LEV SAKHNOVICH

Abstract. We consider the $n \times n$ matrix linear differential systems in the complex plane. We find necessary and sufficient conditions under which these systems have meromorphic fundamental solutions. Using the operator identity method we construct a set of systems which have meromorphic solutions. We prove that the well known operator with the sine kernel generates a class of meromorphic Painlevé type functions. The fifth Painlevé function belongs to this class. Hence we obtain a new and simple proof that the fifth Painlevé function is meromorphic.

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