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UNIVERSAL ZERO PATTERNS FOR SIMULTANEOUS SIMILARITY OF SEVERAL MATRICES

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Abstract. Our field F is algebraically closed. Let M_n be the space of $n \times n$ matrices over F. If $(X_1, \ldots, X_s) \in M_n^s$ and $X_k = \left[x_{ij}^{(k)}\right]$, we say that the triple (i,j,k) labels the entry $x_{ij}^{(k)}$. If P is a collection of labels (i,j,k), then M_P denotes the subspace of M_n^s consisting of $(X_1, \ldots, X_s) \in M_n^s$ such that $x_{ij}^{(k)} = 0$ for all $(i,j,k) \in P$. We present a method of proving that certain patterns P, with $|P| = n(n-1)/2$, are universal in the sense that for every s-tuple $(X_1, \ldots, X_s) \in M_n^s$ there exists $S \in \mathrm{GL}_n(F)$ such that $(SX_1S^{-1}, \ldots, SX_sS^{-1}) \in M_P$. We demonstrate the power of our method on several examples from recent literature.

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