

ON THE CONVERGENCE OF ALUTHGE SEQUENCE

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Abstract. For $0 < \lambda < 1$, the λ -Aluthge sequence $\{\Delta_\lambda^m(X)\}_{m \in \mathbb{N}}$ converges if the nonzero eigenvalues of $X \in \mathbb{C}_{n \times n}$ have distinct moduli, where $\Delta_\lambda(X) := P^\lambda UP^{1-\lambda}$ if $X = UP$ is a polar decomposition of X .

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