

## EXTREMAL MARGINAL TRACIAL STATES IN COUPLED SYSTEMS

GEOFFREY L. PRICE \* AND SHÔICHIRO SAKAI

**Abstract.** Let  $\Gamma$  be the convex set consisting of all states  $\phi$  on the tensor product  $B \otimes B$  of the algebra  $B = M_n(\mathbb{C})$  of all  $n \times n$  matrices over the complex numbers  $\mathbb{C}$  with the property that the restrictions  $\phi|_{B \otimes I}$  and  $\phi|_{I \otimes B}$  are the unique tracial states on  $B \otimes I$  and  $I \otimes B$ . We find necessary and sufficient conditions for such a state, called a marginal tracial state, to be extremal in  $\Gamma$ . We also give a characterization of those extreme points in  $\Gamma$  which are pure states. We conjecture that all extremal marginal tracial states are pure states.

*Mathematics subject classification (2000):* 46L30, 46L06.

*Key words and phrases:* Pure state, trace, marginal tracial state, Schmidt decomposition.

### REFERENCES

- [EK] A. EKERT AND P. L. KNIGHT, *Entangled quantum systems and the Schmidt decomposition*, Am. J. Phys. **63** (1995), 415–423.
- [P1] K. R. PARTHASARATHY, *Extremal quantum states in coupled systems*, Ann. Inst. H. Poincaré **41** (2005), 257–268.
- [P2] K. R. PARTHASARATHY, *On extremal correlations*, J. Stat. Planning and Inf. **103** (2002), 73–80.
- [Sa] S. SAKAI, *C\*-algebras and W\*-algebras*, Springer-Verlag, 1971.
- [Sc] E. SCHMIDT, *Zur theorie der linearen und nichtlinearen integralgleichungen*, Math. Annalen **63** (1906), 433–476.
- [Sz] B. SZ.-NAGY, *Spektraldarstellung linearer transformationen des Hilbertschen raumes*, Springer-Verlag, 1942.
- [T] J. TOMIYAMA, *Applications of Fubini type theorem to the tensor products of C\*-algebras*, Tôhoku Math. J. (2) **19** (1967), 340–344.
- [VW] K. G. H. VOLLBRECHT AND R. F. WERNER, *Why two qubits are special*, J. Math. Phys. **41** (2000), 6772–6782.