

SPHERICAL POTENTIALS OF COMPLEX ORDER IN WEIGHTED GENERALIZED HÖLDER SPACES WITH RADIAL OSCILLATING WEIGHTS

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Abstract. For the spherical potential and hypersingular operators, in general of complex order including the purely imaginary case, there are proved weighted Zygmund type estimates with radial type weights of the Zygmund-Bary-Stechkin class, which may oscillate between power functions. By means of those estimates there are proved boundedness theorems for these operators in weighted generalized Hölder spaces and it is shown that the potential type operator realizes a certain isomorphism within the frameworks of such spaces.

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