

## HIGHER-RANK NUMERICAL RANGES OF UNITARY AND NORMAL MATRICES

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*Abstract.* We verify a conjecture on the structure of higher-rank numerical ranges for a wide class of unitary and normal matrices. Using analytic and geometric techniques, we show precisely how the higher-rank numerical ranges for a generic unitary matrix are given by complex polygons determined by the spectral structure of the matrix. We discuss applications of the results to quantum error correction, specifically to the problem of identification and construction of codes for binary unitary noise models.

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