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SEMI–FREDHOLM SINGULAR INTEGRAL OPERATORS WITH PIECEWISE CONTINUOUS COEFFICIENTS ON WEIGHTED VARIABLE LEBESGUE SPACES ARE FREDHOLM

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Abstract. Suppose Γ is a Carleson Jordan curve with logarithmic whirl points, ϱ is a Khvedelidze weight, $p : \Gamma \to (1, \infty)$ is a continuous function satisfying $|p(\tau) - p(t)| \leqslant -\text{const}/\log|\tau - t|$ for $|\tau - t| \leqslant 1/2$, and $L^{p(\cdot)}(\Gamma, \varrho)$ is a weighted generalized Lebesgue space with variable exponent. We prove that all semi-Fredholm operators in the algebra of singular integral operators with $N \times N$ matrix piecewise continuous coefficients are Fredholm on $L^{p(\cdot)}_N(\Gamma, \varrho)$.

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