

ALGEBRAIC PROPERTIES OF TRUNCATED TOEPLITZ OPERATORS

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Abstract. Compressions of Toeplitz operators to coinvariant subspaces of H^2 are studied. Several characterizations of such operators are obtained; in particular, those of rank one are described. The paper is partly expository. Open questions are raised.

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