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OPTIMIZATION OF THE SPECTRAL RADIUS OF NONNEGATIVE MATRICES

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Abstract. In a recent paper by Axtell, Han, Hershkowitz, and the present authors, one of the main questions that was considered was finding $n\times n$ doubly stochastic matrices P and Q which solve the *multiplicative extremal spectral radius problems* $\min_{S\in\Omega_n}\rho(SA)$ and $\max_{S\in\Omega_n}\rho(SA)$, respectively. Here $A\in\mathbb{R}^{n,n}$ is an arbitrary, but fixed, $n\times n$ nonnegative matrix, $\rho(\cdot)$ is the spectral radius of a matrix, and Ω_n is the set of all $n\times n$ doubly stochastic matrices. It was shown there that the solution to both problems is attained at some permutation matrix. In this paper we consider an additive version of these problems, namely, of solving the *additive extremal spectral radius problems* $\min_{S\in\Omega_n}\rho(S+A)$ and $\max_{S\in\Omega_n}\rho(S+A)$. As a by product of, actually, solutions to more general spectral radius optimization problems, we obtain here that the solution to both additive spectral radius optimization problems is, once again, attained at some permutation matrix. One of the more general spectral radius optimization problems that we consider here is that of replacing the constrains that the optimization be done on the doubly stochastic matrices by the weaker constraint of optimizing just on the $n\times n$ column or row stochastic matrices.

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