

ON THE MULTIPLICITY FUNCTION OF REAL NORMAL OPERATORS

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Abstract. Properties of commuting and non-commuting operators are important in operator theory. In this paper we study the consequences of equations such as $AB = BA^*$ when A and B are real normal operators (A is real if it commutes with a conjugation operator, see [1] or [14]). It is shown that the multiplicity function of A has certain symmetry properties in this case. This generalizes to infinite dimensions some recent results concerning the eigenvalues of certain normal matrices (see [9]).

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