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MARCHENKO EQUATIONS AND NORMING CONSTANTS OF THE MATRIX ZAKHAROV–SHABAT SYSTEM

FRANCESCO DEMONTIS AND CORNELIS VAN DER MEE

Abstract. In this article we derive the Marchenko integral equations for solving the inverse scattering problem for the matrix Zakharov-Shabat system with a potential without symmetry properties and having L^1 entries under a technical hypothesis preventing the accumulation of discrete eigenvalues on the continuous spectrum. We derive additional symmetry properties in the focusing case. The norming constant matrices appearing as parameter matrices in the Marchenko integral kernels are defined and studied without making any assumptions on the Jordan structure of the matrix Zakharov-Shabat Hamiltonian at the discrete eigenvalues.

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