

THE ASYMPTOTIC BEHAVIOR OF FREE ADDITIVE CONVOLUTION

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Abstract. We provide a new proof of the limit theorem for sums of free random variables in a general infinitesimal triangular array. This result was proved by Chistyakov and Götze using subordination functions. Our proof does not depend on subordination, and is close to the approach used in the case of arrays with identically distributed rows [5].

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