

## NUMERICAL RADIUS INEQUALITIES FOR SQUARE-ZERO AND IDEMPOTENT OPERATORS

HWA-LONG GAU, CHIN-YING HUANG AND PEI YUAN WU

*Abstract.* We show that if  $A$  is a square-zero or an idempotent operator on a Hilbert space and  $B$  commutes with  $A$ , then  $w(AB) \leq \min\{w(A)\|B\|, \|A\|w(B)\}$  holds, where  $w(\cdot)$  and  $\|\cdot\|$  denote, respectively, the numerical radius and operator norm of an operator

*Mathematics subject classification (2000):* 47A12.

*Key words and phrases:* Numerical range, numerical radius, square-zero operator, idempotent operator.

### REFERENCES

- [1] K. GUSTAFSON AND D. K. M. RAO, *Numerical Range. The Field of Values of Linear Operators and Matrices*, Springer, New York, 1997.
- [2] P. R. HALMOS, *A Hilbert Space Problem Book*, 2nd ed., Springer, New York, 1981.
- [3] J. A. R. HOLBROOK, *Multiplicative properties of the numerical radius in operator theory*, J. Reine Angew. Math., 237 (1969), 166–174.
- [4] C.-Y. HUANG, *Numerical Radius Inequalities for Square-zero and Idempotent Matrices*, Master thesis, National Chiao Tung Univ., 2005.
- [5] V. MÜLLER, *The numerical radius of a commuting product*, Michigan Math. J., 35 (1988), 255–260.
- [6] D. K. M. RAO, *Rango numerico de operadores conmutativos*, Rev. Colombiana Mat., 27 (1994), 231–233.
- [7] S.-H. TSO AND P. Y. WU, *Matricial ranges of quadratic operators*, Rocky Mountain J. Math., 29 (1999), 1139–1152.
- [8] P. Y. WU, H.-L. GAU AND M. C. TSAI, *Numerical radius inequality for  $C_0$  contractions*, Linear Algebra Appl. (to appear).