

NUMERICAL RADIUS INEQUALITIES FOR SQUARE-ZERO AND IDEMPOTENT OPERATORS

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Abstract. We show that if A is a square-zero or an idempotent operator on a Hilbert space and B commutes with A , then $w(AB) \leq \min\{w(A)\|B\|, \|A\|w(B)\}$ holds, where $w(\cdot)$ and $\|\cdot\|$ denote, respectively, the numerical radius and operator norm of an operator

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