WEYL MATRIX FUNCTIONS AND INVERSE PROBLEMS
FOR DISCRETE DIRAC–TYPE SELF–ADJOINT SYSTEMS:
EXPLICIT AND GENERAL SOLUTIONS

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Abstract. It is shown that the discrete Dirac-type self-adjoint system is equivalent to the block Szegö recurrence. A representation of the fundamental solution is obtained, inverse problems on the interval and semiaxis are solved. A Borg-Marchenko type result is obtained, too. Connections with block Toeplitz matrices are treated

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