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STIELTJES LIKE FUNCTIONS AND INVERSE PROBLEMS FOR SYSTEMS WITH SCHRÖDINGER OPERATOR

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Abstract. A class of scalar Stieltjes like functions is realized as linear-fractional transformations of transfer functions of conservative systems based on a Schrödinger operator T_h in $L_2[a,+\infty)$ with a non-selfadjoint boundary condition. In particular it is shown that any Stieltjes function of this class can be realized in the unique way so that the main operator $\mathbb{A}$ of a system is an accretive $(*)$-extension of a Schrödinger operator T_h. We derive formulas that restore the system uniquely and allow to find the exact value of a non-real parameter h in the definition of T_h as well as a real parameter μ that appears in the construction of the elements of the realizing system. An elaborate investigation of these formulas shows the dynamics of the restored parameters h and μ in terms of the changing free term γ from the integral representation of the realizable function. It turns out that the parametric equations for the restored parameter h represent different circles whose centers and radii are determined by the realizable function. Similarly, the behavior of the restored parameter μ are described by hyperbolas.

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REFERENCES

[1] N. I. AKHIEZER AND I. M. GLAZMAN, *Theory of linear operators*, Pitman Advanced Publishing Program, 1981.
[2] D. ALPAY, I. GOHBERG, M. A. KAASHOEK, A. L. SAKHNOVICH, *Direct and inverse scattering problem for canonical systems with a strictly pseudoexponential potential*, Math. Nachr. 215 (2000), 5–31.
[3] D. ALPAY AND E. R. TSEKANOVSKIĬ, *Interpolation theory in sectorial Stieltjes classes and explicit system solutions*, Lin. Alg. Appl., 314 (2000), 91–136.
[4] YU. M. ARLINSKIĬ, *On regular (*)-extensions and characteristic matrix valued functions of ordinary differential operators*, Boundary value problems for differential operators, Kiev, 3–13, 1980.
[5] YU. ARLINSKIĬ AND E. TSEKANOVSKIĬ. *Regular (*)-extension of unbounded operators, characteristic operator-functions and realization problems of transfer functions of linear systems.* Preprint, VINITI, Dep.-2867, 72p., 1979.
[6] YU. M. ARLINSKIĬ AND E. R. TSEKANOVSKIĬ, *Linear systems with Schrödinger operators and their transfer functions*, Oper. Theory Adv. Appl., 149, 2004, 47–77.
[7] D. AROV, H. DYM, *Strongly regular J-inner matrix-valued functions and inverse problems for canonical systems*, Oper. Theory Adv. Appl., 160, Birkhauser, Basel, (2005), 101–160.
[8] D. AROV, H. DYM, *Direct and inverse problems for differential systems connected with Dirac systems and related factorization problems*, Indiana Univ. Math. J. 54 (2005), no. 6, 1769–1815.
[9] J. A. BALL AND O. J. STAFFANS, *Conservative state-space realizations of dissipative system behaviors*, Integr. Equ. Oper. Theory (Online), Birkhäuser, 2005, DOI 10.1007/s00020-003-1356-3.
[10] H. BART, I. GOHBERG, AND M. A. KAASHOEK, *Minimal Factorizations of Matrix and Operator Functions*, Operator Theory: Advances and Applications, Vol. 1, Birkhäuser, Basel, 1979.
[11] S. V. BELYI AND E. R. TSEKANOVSKIĬ, *Realization theorems for operator-valued R-functions*, Oper. Theory Adv. Appl., 98 (1997), 55–91.

[12] S. V. BELYI AND E. R. TSEKANOVSKIĬ, *On classes of realizable operator-valued R-functions*, Oper. Theory Adv. Appl., 115 (2000), 85–112.
[13] M. S. BRODSKIĬ, *Triangular and Jordan representations of linear operators*, Moscow, Nauka, 1969 (Russian).
[14] V. A. DERKACH, E. R. TSEKANOVSKIĬ, *On characteristic operator-functions of accretive operator colligations*, Ukrainian Math. Dokl., vol. A, no. 8, (1981), 16–20 (Ukrainian).
[15] I. DOVZHENKO AND E. R. TSEKANOVSKIĬ, *Classes of Stieltjes operator-functions and their conservative realizations*, Dokl. Akad. Nauk SSSR, 311 no. 1 (1990), 18–22.
[16] F. GESZTESY AND E. R. TSEKANOVSKIĬ, *On matrix-valued Herglotz functions*, Math. Nachr., 218 (2000), 61–138.
[17] F. GESZTESY, N. J. KALTON, K. A. MAKAROV, E. TSEKANOVSKIĬ, *Some Applications of Operator-Valued Herglotz Functions*, Operator Theory: Advances and Applications, **123**, Birkhäuser, Basel, (2001), 271–321.
[18] S. KHRUSHCHEV, *Spectral Singularities of dissipative Schrödinger operator with rapidly decreasing potential*, Indiana Univ. Math. J., 33 no. 4, (1984), 613–638.
[19] I. S. KAC AND M. G. KREIN, *R-functions–analytic functions mapping the upper halfplane into itself*, Amer. Math. Soc. Transl. (2) **103**, 1–18 (1974).
[20] T. KATO, *Perturbation Theory for Linear Operators*, Springer-Verlag, 1966 ath. Dokl. **19**, no. 5, 1131–1134 (1978).
[21] B. M. LEVITAN, *Inverse Sturm-Liouville Problems*, Moscow, Nauka, 1984 (Russian). [English translation: VNU Science Press, Utrecht, 1987]
[22] M. S. LIVŠIC, *Operators, oscillations, waves*, Moscow, Nauka, 1966 (Russian).
[23] M. A. NAIMARKN, *Linear Differential Operators II*, F. Ungar Publ., New York, 1968.
[24] O. J. STAFFANS, *Passive and conservative continuous time impedance and scattering systems, Part I: Well posed systems*, Math. Control Signals Systems, 15, (2002), 291–315.
[25] E. R. TSEKANOVSKIĬ, *Accretive extensions and problems on Stieltjes operator-valued functions realizations*, Oper. Theory Adv. Appl., 59 (1992), 328–347.
[26] E. R. TSEKANOVSKIĬ. *Characteristic function and sectorial boundary value problems*, Investigation on geometry and math. analysis, Novosibirsk, **7**, (1987), 180–194.
[27] E. R. TSEKANOVSKIĬ AND YU. L. SHMUL'YAN, *The theory of bi-extensions of operators on rigged Hilbert spaces. Unbounded operator colligations and characteristic functions*, Russ. Math. Surv., 32 (1977), 73–131.
[28] V. A. YURKO, *Inverse problems for differential operators*, Saratv State University Publ., Saratov, 1989 (in Russian).