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DAVIS–WIELANDT SHELLS OF OPERATORS

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Abstract. Basic properties of Davis-Wielandt shells are presented. Conditions on two operators A and B with the same Davis-Wielandt shells are analyzed. Special attention is given to the case when B is a compression of A, and when $B = A^*$, A^t, or $(A^*)^t$, where A^t is the transpose of A with respect to an orthonormal basis. The results are used to study the point spectrum, approximate point spectrum, and residual spectrum of the sum of two operators. Relation between the geometrical properties of the Davis-Wielandt shells and algebraic properties of operators are obtained. Complete descriptions of the Davis-Wielandt shells are given for several classes of operators.

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