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ON THE SPECTRA OF SOME TOEPLITZ AND WIENER–HOPF OPERATORS WITH ALMOST PERIODIC MATRIX SYMBOLS

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Abstract. Constructive invertibilty criteria for Toeplitz and Wiener-Hopf operators with matrix almost periodic symbols in general are not known. Even in the case of 2×2 triangular symbols definite results are available only under some, rather restrictive additional requirements on the entries of those symbols. We show, however, that for certain symbols such additional requirements allow one to go one step further and actually describe the (essential) spectra of the operators in question. This description shows in particular that the number of connected components of the spectrum can be arbitrarily large — a striking difference with the scalar situation.

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