

ON THE ABSOLUTELY CONTINUOUS SPECTRUM OF STURM-LIOUVILLE OPERATORS WITH APPLICATIONS TO RADIAL QUANTUM TREES

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Abstract. We consider standard subordinacy theory for general Sturm–Liouville operators and give criteria when boundedness of solutions implies that no subordinate solutions exist. As applications, we prove a Weidmann-type result for general Sturm–Liouville operators and investigate the absolutely continuous spectrum of radially symmetric quantum trees.

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