

POLYNOMIAL MATRICES WITH HERMITIAN COEFFICIENTS AND A GENERALIZATION OF THE ENESTRÖM–KAKEYA THEOREM

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Abstract. Polynomial matrices $G(z) = Iz^m - \sum C_i z^i$ with hermitian coefficients C_i are studied. The assumption $\sum |C_i| \leq I$ implies that the characteristic values of $G(z)$ lie in the closed unit disc. The characteristic values of modulus one are roots of unity. An extension of the Eneström–Kakeya theorem is proved and a stability criterion for a system of difference equations is given.

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