Operators and Matrices
Volume 2, Number 4 (2008), 455–463

A NEW CHARACTERIZATION OF THE CLOSURE OF THE $(\mathscr{U}+\mathscr{K})$–ORBIT OF CERTAIN ESSENTIALLY NORMAL OPERATORS

FAHUI ZHAI AND JUNJIE ZHAO

Abstract. The $(\mathscr{U}+\mathscr{K})$-orbit of a Hilbert space operator T is defined as $(\mathscr{U}+\mathscr{K})(T)=\{R^{-1}TR : R \text{ invertible of the form unitary plus compact}\}$. In this paper, we show that certain essentially normal operator with the same spectral picture as an essentially normal injective unilateral weighted operator generates the same closure of $(\mathscr{U}+\mathscr{K})$-orbit.

Mathematics subject classification (2000): 47B37, 47B20.
Keywords and phrases: $(\mathscr{U}+\mathscr{K})$-orbit of operator; essentially normal operator; spectral picture; injective unilateral weighted operator.

REFERENCES

[1] F. A. AL-MUSALLAM, *An upper estimate for the distance to essentially G_1 operators*, Ph. D. thesis., Arizona state University (1989).
[2] I. D. BERG AND K. R. DAVIDSON, *Almost Community Matrices and a quantitative of the Brown-Dong las-Fillmore theorem*, Acta Math., **166** (1991), 121–161.
[3] J. B. COMWAY, *A course in functional analysis*, 2nd ed., Springer-Verlag, 1990.
[4] M. DOSTÁL, *Closures of $(\mathscr{U}+\mathscr{K})$-orbits of essentially normal models*, PHD. thesis, University of Alberta in Canada, 1999.
[5] P. S. GUINAND AND L. W. MARCOUX, *Between the unitary and similarity orbits of normal operators*, Pacific J. Math. **159(2)** (1993), 299–334.
[6] P. S. GUINAND AND L. W. MARCOUX, *On the $(\mathscr{U}+\mathscr{K})$-orbits of certain weighted shifts*, Integral Equations and Operator Theory, **17** (1993), 516–543.
[7] P. R. HALMOS, *A Hilbert Space Problem Book*, Van Nostrand-Reinhold, New York, (1967).
[8] D. A. HERRERO, *Approximation of Hilbert Space Operators.*, Vol. I,. (Research Notes in Math 224), London-Boston-Melbourne: Pitman Books Ltd., (1982).
[9] D. A. HERRERO, *A trace obstruction to approximation by block diagonal nilpotent*, American Journal of mathematics, **108** (1986), 516–543.
[10] H. HARTOGS AND A. ROSENTHAL, *über fdgen analystischer funktionen*, Math, Ann., Vol. 104, (1931), 215–223.
[11] Y. JI AND J. LI, *The quasiapproximate $(u+k)$-invariant of essentially normal operators*, Integral equations and operator theory, 50, (2004), 255–278.
[12] Y. Q. JI, C. L. JIANG AND Z. Y. WANG, *The $(\mathscr{U}+\mathscr{K})$-orbit of essentially normals and compact perturbation of strongly irreducible operators*, Functional analysis in China volume 356 of Mathematics and its applications. Dordrecht; Boston; Kluwer Academic Publishers, 1996.
[13] C. JIANG AND Z. WANG, *Strongly Irreducible Operators on Hilbert Spaces*, Pitman Research Notes in Mathematics Series 389, Addison-Wesley-Longman Company, 1998.
[14] L. W. MARCOUX, *The closure of the $(U+K)$-orbit of shift-like operators*, Indiana Univ. Math. J., **41** (1992), 1211–1223.
[15] L. W. MARCOUX, *A survey of $(\mathscr{U}+\mathscr{K})$-orbits*, Operator theory and Banach algebra, Proceeding of International Conference in Analysis, Rabat, Moroco, April 12–14, (1999), 91–115.
[16] F. H. ZHAI, *The closures of $(\mathscr{U}+\mathscr{K})$-orbits of class essentially normal operators*, Houston. J. Math. **30** (2004), 1177–1194.

Paper OAM-02-28