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A NOTE ON ANISOTROPIC POTENTIALS ASSOCIATED WITH THE LAPLACE–BESSEL DIFFERENTIAL OPERATOR

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Abstract. In this note the anisotropic maximal operator and anisotropic Riesz potentials generated by the generalized shift operator are investigated in the anisotropic B-Morrey space $L_{p,\lambda,\gamma}(\mathbb{R}^n_{k,+})$. We prove that the anisotropic B-maximal operator M_γ is bounded on the anisotropic B-Morrey space $L_{p,\lambda,\gamma}(\mathbb{R}^n_{k,+})$. Also the anisotropic B-Riesz potential R^α_γ is bounded from the anisotropic B-Morrey spaces $L_{p,\lambda,\gamma}(\mathbb{R}^n_{k,+})$ to $L_{q,\lambda,\gamma}(\mathbb{R}^n_{k,+})$ if and only if $1/p-1/q=\alpha/(|a|+(a,\gamma)-\lambda)$ and $1<p<(|a|+(a,\gamma)-\lambda)/\alpha$, and its modified version $\widetilde{R}^\alpha_\gamma$ is bounded from the anisotropic B-Morrey space to the anisotropic B-BMO space. Furthermore, we obtain some imbedding relations between the space $L_{p,\lambda,\gamma}(\mathbb{R}^n_{k,+})$ and the anisotropic B-Stummel-Kato class $S_{p,\theta,\gamma}(\mathbb{R}^n_{k,+})$.

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