

ESTIMATES OF INVERSES OF MULTIVARIABLE TOEPLITZ MATRICES

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Abstract. The Gohberg-Semencul formula provides a formula for the inverse of a Toeplitz matrix based on the entries in the first and last columns of the inverse, under certain nonsingularity conditions. In this paper we study similar formulas for multivariable Toeplitz matrices, and we show that in the positive definite case these expressions provide upper bounds for the inverse in the Loewner order. Some numerical experiments regarding the proximity of the estimate are included.

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