Operators and Matrices
Volume 2, Number 4 (2008), 525–541

ASYMPTOTIC PSEUDOMODES OF TOEPLITZ MATRICES

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Abstract. Questions in probability and statistical physics lead to the problem of finding the eigenvectors associated with the extreme eigenvalues of Toeplitz matrices generated by Fisher-Hartwig symbols. We here simplify the problem and consider pseudomodes instead of eigenvectors. This replacement allows us to treat fairly general symbols, which are far beyond Fisher-Hartwig symbols. Our main result delivers a variety of concrete unit vectors x_n such that if $T_n(a)$ is the $n \times n$ truncation of the infinite Toeplitz matrix generated by a function $a \in L^1$ satisfying mild additional conditions and λ is in the range of this function, then $\|T_n(a)x_n - \lambda x_n\| \to 0$.

Mathematics subject classification (2000): 47B35, 15A18, 41A80, 46N30.
Keywords and phrases: Toeplitz matrix, Fisher-Hartwig symbol, eigenvector, pseudomode, fractional Brownian motion.

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