

## INVERSE PROBLEMS FOR STURM-LIOUVILLE OPERATORS ON GRAPHS WITH A CYCLE

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*Abstract.* An inverse spectral problem is studied for second-order differential operators on graphs with a cycle and with standard matching conditions in the internal vertex. A uniqueness theorem is proved, and a constructive procedure for the solution is provided.

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