

UNBOUNDED OPERATORS COMMUTING WITH RESTRICTED BACKWARD SHIFTS

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Abstract. The closed densely defined operators on a proper invariant subspace of the backward shift that commute with the restricted backward shift are shown to be coanalytic Toeplitz operators induced by functions in the Nevanlinna class. The result can be interpreted as a kind of commutant lifting theorem for unbounded operators. It extends, and in a certain sense completes, earlier work of Daniel Suárez.

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