

CONSERVATIVE REALIZATIONS OF THE FUNCTIONS ASSOCIATED WITH SCHUR'S ALGORITHM FOR THE SCHUR CLASS OPERATOR-VALUED FUNCTION

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Abstract. Let \mathfrak{M} and \mathfrak{N} be separable Hilbert spaces and let $\Theta(\lambda)$ be a function from the Schur class $\mathbf{S}(\mathfrak{M}, \mathfrak{N})$ of contractive functions holomorphic on the unit disk. The operator generalization of the classical Schur algorithm associates with Θ the sequence of contractions (the Schur parameters of Θ) $\Gamma_0 = \Theta(0) \in \mathbf{L}(\mathfrak{M}, \mathfrak{N})$, $\Gamma_n \in \mathbf{L}(\mathfrak{D}_{\Gamma_{n-1}}, \mathfrak{D}_{\Gamma_{n-1}}^*)$ and the sequence of functions $\Theta_0 = \Theta$, $\Theta_n \in \mathbf{S}(\mathfrak{D}_{\Gamma_n}, \mathfrak{D}_{\Gamma_n}^*)$ $n = 1, \dots$ (the Schur iterates of Θ) connected by the relations

$$\Gamma_n = \Theta_n(0), \Theta_n(\lambda) = \Gamma_n + \lambda D_{\Gamma_n} \Theta_{n+1}(\lambda) (I + \lambda \Gamma_n^* \Theta_{n+1}(\lambda))^{-1} D_{\Gamma_n}, |\lambda| < 1.$$

It is well known that the function $\Theta(\lambda) \in \mathbf{S}(\mathfrak{M}, \mathfrak{N})$ can be realized as the transfer function

$$\Theta(\lambda) = D + \lambda C(I - \lambda A)^{-1} B$$

of a linear conservative and simple discrete time-invariant system $\tau = \left\{ \begin{bmatrix} D & C \\ B & A \end{bmatrix}; \mathfrak{M}, \mathfrak{N}, \mathfrak{H} \right\}$ with the state space \mathfrak{H} and the input and output spaces \mathfrak{M} and \mathfrak{N} , respectively.

In this paper we give a construction of conservative and simple realizations of the Schur iterates Θ_n by means of the conservative and simple realization of Θ .

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