

AN ELEMENTARY PROOF OF VOICULESCU'S ASYMPTOTIC FREENESS FOR RANDOM UNITARY MATRICES

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Abstract. D. Voiculescu [2] proved that a standard family of independent random unitary $k \times k$ matrices and a constant $k \times k$ unitary matrix is asymptotically free as $k \rightarrow \infty$. This result was a key ingredient in Voiculescu's proof [3] that his free entropy is additive when the variables are free. In this paper, we give a very elementary proof of a more detailed version of this result [2]. We have not yet recaptured Voiculescu's strengthened version [4].

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