PRESERVERS FOR NORMS OF LIE PRODUCT

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Abstract. Let $\| \cdot \|$ be a unitary similarity invariant norm on the set M_n of $n \times n$ complex matrices. A description is obtained for surjective maps ϕ on M_n satisfying $\|AB - BA\| = \|\phi(A)\phi(B) - \phi(B)\phi(A)\|$ for all $A, B \in M_n$. The general theorem covers the special cases when the norm is one of the Schatten p-norms, the Ky-Fan k-norms, or the k-numerical radii.

Mathematics subject classification (2000): Primary 15A60, 46B04.
Keywords and phrases: Lie product, unitarily invariant and unitary similarity invariant norms.

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