

RECONSTRUCTION OF THE TRANSMISSION COEFFICIENT FOR STEPLIKE FINITE-GAP BACKGROUNDS

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Abstract. We consider scattering theory for one-dimensional Jacobi operators with respect to steplike quasi-periodic finite-gap backgrounds and show how the transmission coefficient can be reconstructed from minimal scattering data. This generalizes the Poisson–Jensen formula for the classical constant background case.

Mathematics subject classification (2000): Primary 30E20, 30F30; Secondary 34L25, 47B36.

Keywords and phrases: Jacobi operators, scattering theory, periodic, Abelian integrals.

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