

## RECONSTRUCTION OF THE TRANSMISSION COEFFICIENT FOR STEPLIKE FINITE-GAP BACKGROUNDS

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*Abstract.* We consider scattering theory for one-dimensional Jacobi operators with respect to steplike quasi-periodic finite-gap backgrounds and show how the transmission coefficient can be reconstructed from minimal scattering data. This generalizes the Poisson–Jensen formula for the classical constant background case.

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