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# ON THE POLAR DECOMPOSITION OF THE DUGGAL TRANSFORMATION AND RELATED RESULTS

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*Abstract.* Let $T = U|T|$ be the polar decomposition of a bounded operator $T$ on a Hilbert space. The transformation $\Delta(T) = |T|^{1/2}U|T|^{1/2}$ is called the Aluthge transformation, and $\Gamma(T) = |T|U$ is called the Duggal transformation of $T$. We discuss Aluthge transformation and Duggal transformation of binormal operators and centered operators. We obtain results about the polar decomposition of Duggal transformation. We give necessary and sufficient conditions for $\Gamma(T)$ to have the polar decomposition $\Gamma(T) = \Gamma(U)|\Gamma(T)|$. As a consequence we get $\Gamma(T) = \Gamma(U)|\Gamma(T)|$ to be the polar decomposition of $\Gamma(T)$ if $T$ is binormal.

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