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HYPERINVARIANT, CHARACTERISTIC AND MARKED SUBSPACES

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Abstract. Let V be a finite dimensional vector space over a field K and f a K-endomorphism of V. In this paper we study three types of f-invariant subspaces, namely hyperinvariant subspaces, which are invariant under all endomorphisms of V that commute with f, characteristic subspaces, which remain fixed under all automorphisms of V that commute with f, and marked subspaces, which have a Jordan basis (with respect to $f_{|X}$) that can be extended to a Jordan basis of V. We show that a subspace is hyperinvariant if and only if it is characteristic and marked. If K has more than two elements then each characteristic subspace is hyperinvariant.

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