

## NUMERICAL RANGES OF RESTRICTED SHIFTS AND UNITARY DILATIONS

ISABELLE CHALENDAR, PAMELA GORKIN AND JONATHAN R. PARTINGTON

*Abstract.* This paper considers the restricted shift operator associated with an infinite Blaschke product, expressing the closure of its numerical range as the intersection of the closures of the numerical ranges of a parametrized family of unitary dilations (or, equivalently, unitary perturbations of a modified restricted shift). The techniques used are based on interpolation and Clark measures. The results generalize known theorems for numerical ranges of matrices associated with finite-dimensional Blaschke products, which can be expressed geometrically in terms of the Poncelet property.

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