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BOUNDARY RIGIDITY FOR SOME CLASSES OF MEROMORPHIC FUNCTIONS

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Abstract. Let f be a function meromorphic on the open unit disk $\mathbb{D}$, with angular boundary limits bounded by one in modulus almost everywhere on the unit circle. We give sufficient conditions in terms of boundary asymptotics at finitely many points on the unit circle $\mathbb{T}$ for f to be a ratio of two finite Blaschke products. A necessary condition is that f has finitely many poles in $\mathbb{D}$, i.e., that f is a generalized Schur function. Similar rigidity statements are presented for generalized Carathéodory and generalized Nevanlinna functions.

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