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STRUCTURED DECOMPOSITIONS FOR MATRIX TRIPLES: SVD–LIKE CONCEPTS FOR STRUCTURED MATRICES

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Abstract. Canonical forms for matrix triples $(A,G,\hat{G})$, where A is arbitrary rectangular and G, $\hat{G}$ are either real symmetric or skew symmetric, or complex Hermitian or skew Hermitian, are derived. These forms generalize classical singular value decompositions. In [1] a similar canonical form has been obtained for the complex case. In this paper, we provide an alternative proof for the complex case which is based on the construction of a staircase-like form with the help of a structured QR-like decomposition. This approach allows generalization to the real case.

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