

## ON BOUNDS FOR DISCRETE SEMIGROUPS

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*Abstract.* The main result of this note is extension on the infinite dimension of the following known result for finite matrices: while the spectral radius  $\rho(T)$  gives only asymptotic decay estimates, the solution  $X$  of the *discrete Lyapunov equation*  $X - T^*XT = BB^*$  yields rigorous bounds. We also present a new upper bound for the norm of the solution  $X$  in the matrix case which depends on the structure of the right hand side. The new bound shows that the structure of  $B$  can greatly influence  $\|X\|$ .

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