

ESSENTIALLY HERMITIAN MATRICES AND INCLUSION RELATIONS OF C -NUMERICAL RANGES

WAI-SHUN CHEUNG

Abstract. Let \mathbf{M} denote the set of all $n \times n$ complex matrices and \mathbf{M}_n^0 denote the set of $n \times n$ matrices with trace 0. For any $C \in \mathbf{M}_n^0$, there exists a maximal $v(C) \geq 0$ such that

$$v(C)W_D(A) \subseteq \|D\|_F W_C(A)$$

whenever $D \in \mathbf{M}_n^0$ and $A \in \mathbf{M}_n$. Here $W_C(A)$ denotes the C -numerical range of A and $\|D\|_F$ denotes the Frobenius norm of D . Moreover $v(C) = 0$ if and only if C is essentially hermitian.

To prove the above result, we have obtained a new characterisation of essentially hermitian matrices.

Mathematics subject classification (2000): 15A57, 47A12.

Keywords and phrases: essentially hermitian matrix, numerical range.

REFERENCES

- [1] Y. H. AU-YEUNG AND N. K. TSING, *A conjecture of Marcus on the generalized numerical range*, Linear and Multilinear Algebra, **14** (1983), 235–239.
- [2] W. S. CHEUNG, *Some Geometrical Aspects Of and Inclusion Relations For Generalized Numerical Ranges*, M. Phil Thesis, The University of Hong Kong, 1996.
- [3] W. S. CHEUNG AND N. K. TSING, *The C -numerical range of matrices is star-shaped*, Linear and Multilinear Algebra, **41**, 3 (1996), 245–250.
- [4] M. GOLDBERG AND E. G. STRAUS, *Elementary inclusion relations for generalized numerical ranges*, Lin. Alg. Appl., **18** (1977), 1–18.
- [5] R. HORN AND C. R. JOHNSON, *Topics in Matrix Analysis*, Cambridge University Press, Cambridge, 1991.
- [6] C.-K. LI, *C -numerical ranges and C -numerical radii*, Linear and Multilinear Algebra, **37** (1994), 51–82.
- [7] C.-K. LI AND N. K. TSING, *Matrices with circular symmetry on their unitary similarity orbits and C -numerical ranges*, Proc. Amer. Math. Soc., **111** (1991), 19–28.
- [8] Y. T. POON, *Another proof of a result of Westwick*, Linear and Multilinear Algebra, **9** (1980), 35–37.
- [9] N. K. TSING, *The constrained bilinear form and the C -numerical range*, Linear Algebra and Its Applications, **56** (1984), 195–206.
- [10] R. WESTWICK, *A theorem on numerical ranges*, Linear and Multilinear Algebra, **2** (1975), 311–315.