

ASYMPTOTIC BEHAVIOR OF GELFAND–NAIMARK DECOMPOSITION

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Abstract. Let $X = L\sigma U$ be the Gelfand-Naimark decomposition of $X \in GL_n(\mathbb{C})$, where L is unit lower triangular, σ is a permutation matrix, and U is upper triangular. Call $u(X) := \text{diag } U$ the u -component of X . We show that in a Zariski dense open subset of the ω -orbit of certain Bruhat decomposition,

$$\lim_{m \rightarrow \infty} |u(X^m)|^{1/m} = \text{diag}(|\lambda_{\omega(1)}|, \dots, |\lambda_{\omega(n)}|).$$

The other situations where $\lim_{m \rightarrow \infty} |u(X^m)|^{1/m}$ converge to different limits or diverge are also discussed.

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