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ASYMPTOTIC BEHAVIOR OF GELFAND–NAIMARK DECOMPOSITION

HUAJUN HUANG

Abstract. Let $X = L\sigma U$ be the Gelfand-Naimark decomposition of $X \in \mathrm{GL}_n(\mathbb{C})$, where L is unit lower triangular, σ is a permutation matrix, and U is upper triangular. Call $u(X) := \mathrm{diag}\, U$ the u-component of X. We show that in a Zariski dense open subset of the ω-orbit of certain Bruhat decomposition,

$$\lim_{m\to\infty} |u(X^m)|^{1/m} = \mathrm{diag}\,(|\lambda_{\omega(1)}|, \cdots, |\lambda_{\omega(n)}|).$$

The other situations where $\lim_{m\to\infty} |u(X^m)|^{1/m}$ converge to different limits or diverge are also discussed.

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REFERENCES

[1] R. G. BARTLE AND D. R. SHERBERT, *Introduction to Real Analysis*, Wiley, 1999.
[2] R. GOODMAN AND N. R. WALLACH, *Representations and Invariants of the Classical Groups*, Cambridge University Press, 2000.
[3] R. A. HORN AND C. R. JOHNSON, *Topics in Matrix Analysis*, Cambridge Univ. Press, 1991.
[4] H. HUANG AND T. Y. TAM, *An asymptotic behavior of QR decomposition*, Linear Algebra and Its Applications, **424** (2007) 96–107.
[5] H. HUANG AND T. Y. TAM, *On Gelfand-Naimark decomposition of a nonsingular matrix*, Linear and Multilinear Algebra (to appear).
[6] H. HUANG AND T. Y. TAM, *On the QR iterations of real matrices*, Linear Algebra and Its Applications, **408** (2005) 161–176.
[7] H. HUANG AND T. Y. TAM, *Some asymptotic behaviors associated with matrix decomposition*, International J. of Information & Systems Sciences, (2008) 148–159.
[8] E. TYRTYSHNIKOV, *Matrix Bruhat decompositions with a remark on the QR (GR) algorithm*, Linear Algebra and Its Applications, **250** (1997) 61–68.
[9] T. YAMAMOTO, Extreme values of the roots of matrices, J. Math. Soc. Japan, **19** (1967) 171–178.

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