SUMS OF NEVANLINNA FUNCTIONS AND
DIFFERENTIAL EQUATIONS ON STAR–SHAPED GRAPHS

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Abstract. Additive decompositions of a meromorphic function give rise to quotient representations of a particular form. We raise the question which quotient representations of a given function arise in this way. This question is answered by means of two characterizations via different terms. We pay particular attention to functions belonging to various subclasses of the Nevanlinna class of functions with nonnegative imaginary part throughout the upper half-plane. Our results lead to some direct and inverse spectral theorems for systems of strings or systems of Sturm-Liouville equations supported on a star-shaped graph.

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