

STUDY OF A DIFFERENTIAL OPERATOR OF HEUN TYPE ARISING IN FLUID DYNAMICS

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Abstract. The paper studies the non-selfadjoint linear differential operator

$$Ly = \frac{d}{dt} \left((1 - a \cos t)y + b \sin t \frac{dy}{dt} \right)$$

acting in the Hilbert space $L^2(-\pi, \pi)$ that originated from a steady state stability problem in fluid dynamics. The operator L is of Heun type and involves two parameters a, b related to the hydrostatic pressure and capillary properties of the fluid. The results concern (1) the properties of functions in the domain of definition of L , (2) conditions on a, b for the linear span of the Fourier basis $\{e^{int}\}$ to be core of L , and (3) the matrix representation of the reduced resolvent of L in the Fourier basis. In particular, it is shown that the reduced resolvent is compact and of trace class \mathcal{S}_1 .

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