

## STUDY OF A DIFFERENTIAL OPERATOR OF HEUN TYPE ARISING IN FLUID DYNAMICS

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*Abstract.* The paper studies the non-selfadjoint linear differential operator

$$Ly = \frac{d}{dt} \left( (1 - a \cos t)y + b \sin t \frac{dy}{dt} \right)$$

acting in the Hilbert space  $L^2(-\pi, \pi)$  that originated from a steady state stability problem in fluid dynamics. The operator  $L$  is of Heun type and involves two parameters  $a, b$  related to the hydrostatic pressure and capillary properties of the fluid. The results concern (1) the properties of functions in the domain of definition of  $L$ , (2) conditions on  $a, b$  for the linear span of the Fourier basis  $\{e^{int}\}$  to be core of  $L$ , and (3) the matrix representation of the reduced resolvent of  $L$  in the Fourier basis. In particular, it is shown that the reduced resolvent is compact and of trace class  $\mathcal{S}_1$ .

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