STUDY OF A DIFFERENTIAL OPERATOR OF HEUN TYPE ARISING IN FLUID DYNAMICS

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Abstract. The paper studies the non-selfadjoint linear differential operator

\[ Ly = \frac{d}{dt} \left( (1 - a \cos t)y + b \sin t \frac{dy}{dt} \right) \]

acting in the Hilbert space \( L^2(-\pi, \pi) \) that originated from a steady state stability problem in fluid dynamics. The operator \( L \) is of Heun type and involves two parameters \( a, b \) related to the hydrostatic pressure and capillary properties of the fluid. The results concern (1) the properties of functions in the domain of definition of \( L \), (2) conditions on \( a, b \) for the linear span of the Fourier basis \( \{e^{int}\} \) to be core of \( L \), and (3) the matrix representation of the reduced resolvent of \( L \) in the Fourier basis. In particular, it is shown that the reduced resolvent is compact and of trace class \( \mathcal{S}_1 \).


Keywords and phrases: internal singularity, Heun equation, reduced resolvent, core of an operator, Schatten class.

REFERENCES