Operators and Matrices
Volume 3, Number 4 (2009), 557–569

SUPPLEMENTARY DIFFERENCE SETS WITH SYMMETRY FOR HADAMARD MATRICES

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Abstract. An overview of the known supplementary difference sets (SDSs) (A_i), $1 \leqslant i \leqslant 4$, with parameters $(n;k_i;\lambda)$, $k_i = |A_i|$, where each A_i is either symmetric or skew and $\sum k_i = n+\lambda$ is given. Five new Williamson matrices over the elementary abelian groups of order 5^2, 3^3 and 7^2 are constructed. New examples of skew Hadamard matrices of order $4n$ for $n = 47, 61, 127$ are presented. The last of these is obtained from a $(127, 57, 76)$ difference family that we have constructed. An old non-published example of G-matrices of order 37 is also included.

Mathematics subject classification (2000): 05B20, 05B30.
Keywords and phrases: Supplementary difference sets, Hadamard matrices, Williamson matrices, Goethals–Seidel array.

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