

## SUPPLEMENTARY DIFFERENCE SETS WITH SYMMETRY FOR HADAMARD MATRICES

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*Abstract.* An overview of the known supplementary difference sets (SDSs)  $(A_i)$ ,  $1 \leq i \leq 4$ , with parameters  $(n; k_i; \lambda)$ ,  $k_i = |A_i|$ , where each  $A_i$  is either symmetric or skew and  $\sum k_i = n + \lambda$  is given. Five new Williamson matrices over the elementary abelian groups of order  $5^2$ ,  $3^3$  and  $7^2$  are constructed. New examples of skew Hadamard matrices of order  $4n$  for  $n = 47, 61, 127$  are presented. The last of these is obtained from a  $(127, 57, 76)$  difference family that we have constructed. An old non-published example of G-matrices of order 37 is also included.

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