

AGLER INTERPOLATION FAMILIES OF KERNELS

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Abstract. An abstract Pick interpolation theorem for a family of positive semi-definite kernels on a set X is formulated. The result complements those in [Ag] and [AM02] and will subsequently be applied to Pick interpolation on distinguished varieties [JKM].

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