

THE KATO DECOMPOSITION OF QUASI-FREDHOLM RELATIONS

J.-PH. LABROUSSE, A. SANDOVICI, H. S. V. DE SNOO
AND H. WINKLER

Abstract. Quasi-Fredholm relations of degree $d \in \mathbb{N}$ in Hilbert spaces are defined in terms of conditions on their ranges and kernels. They are completely characterized in terms of an algebraic decomposition with a quasi-Fredholm relation of degree 0 and a nilpotent operator of degree d . The adjoint of a quasi-Fredholm relation of degree $d \in \mathbb{N}$ is shown to be quasi-Fredholm relation of degree $d \in \mathbb{N}$. The class of quasi-Fredholm relations contains the semi-Fredholm relations. Earlier results for quasi-Fredholm operators and semi-Fredholm operators are included.

Mathematics subject classification (2000): Primary 47A57, 47B25; Secondary 47A55, 47B65.

Keywords and phrases: Quasi-Fredholm relation, semi-Fredholm relation, Kato decomposition.

REFERENCES

- [1] R. W. CROSS, *Multivalued linear operators*, Marcel Dekker, New York, 1998.
- [2] A. DIJKSMA AND H. S. V. DE SNOO, *Symmetric and selfadjoint relations in Kreĭn spaces I*, Ann. Acad. Sci. Fenn. Ser. A I Math., 12 (1987), 199–216.
- [3] P. A. FILLMORE AND J. P. WILLIAMS, *On operator ranges*, Adv. Math., 7 (1971), 254–281.
- [4] I. C. GOHBERG AND M. G. KREĪN, *The basic propositions on defect numbers, root numbers and indices of linear operators*, Uspekhi Mat. Nauk., 12 (1957), 43–118 (Russian) [English translation: Transl. Amer. Math. Soc. (2), 13 (1960), 185–264].
- [5] S. HASSI, Z. SEBESTYÉN, H. S. V. DE SNOO, AND F. H. SZAFRANIEC, *A canonical decomposition for linear operators and linear relations*, Acta Math. Hungar., 115 (2007), 281–307.
- [6] S. HASSI, H. S. V. DE SNOO, AND F. H. SZAFRANIEC, *Componentwise and canonical decompositions of linear relations*, Dissertationes Mathematicae, 465 (2009), 59 pp.
- [7] T. KATO, *Perturbation theory for nullity, deficiency, and other quantities of linear operators*, J. d'Anal. Math., 6 (1958), 261–322.
- [8] T. KATO, *Perturbation theory for linear operators*, Springer-Verlag, Berlin, 1980.
- [9] J.-PH. LABROUSSE, *Les opérateurs quasi Fredholm: une généralisation des opérateurs semi Fredholm*, Rend. Circ. Mat. Palermo (2), 29 (1980), 161–258.
- [10] J.-PH. LABROUSSE, *Idempotent linear relations*, in Spectral Theory and Its Applications, Theta, 2003, 129–149.
- [11] J.-PH. LABROUSSE, A. SANDOVICI, H. S. V. DE SNOO, AND H. WINKLER, *Quasi-Fredholm relations in Hilbert spaces*, Universitatea din Bacau Studii si Cercetari Stiintifice, Ser. Mat., 16 (2006), 93–106
- [12] M. MBEKTHA, *Sur l'unicité de la décomposition de Kato généralisée*, Acta Sci. Math. (Szeged), 54 (1990), 367–377.
- [13] Y. MEZROUI, *Le complété des opérateurs fermés à domaine dense pour la métrique du gap*, J. Operator Theory, 41 (1999), 69–92.
- [14] V. MÜLLER, *On the Kato decomposition of quasi-Fredholm operators and B-Fredholm operators*, Proc. Workshop Geometry in Functional Analysis, Erwin Schrödinger Institute, Wien, 2000.
- [15] V. MÜLLER AND M. MBEKTHA, *On the axiomatic theory of spectrum*, Studia Math., 119 (1996), 129–147.
- [16] V. RAKOČEVIĆ, *Semi-Browder operators and perturbations*, Studia Math., 122 (1997), 131–137.

- [17] A. SANDOVICI, H. S. V. DE SNOO, AND H. WINKLER, *The structure of linear relations in Euclidean spaces*, Lin. Alg. Appl., 397 (2005), 141–169.
- [18] A. SANDOVICI, H. S. V. DE SNOO, AND H. WINKLER, *Ascent, descent, nullity, defect, and related notions for linear relations in linear spaces*, Lin. Alg. Appl., 423 (2007), 456–497.