Operators
and
Matrices
Volume 4, Number 1 (2010), 103–118

CONTRACTIBILITY OF THE MAXIMAL IDEAL SPACE OF ALGEBRAS OF MEASURES IN A HALF–SPACE

AMOL SASANE

Abstract. Let $\mathbb{H}^{[n]}$ be the canonical half space in $\mathbb{R}^n$, that is,

$$\mathbb{H}^{[n]} = \{(t_1,\dots,t_n) \in \mathbb{R}^n \setminus \{0\} \mid \forall j,\ [t_j \neq 0 \text{ and } t_1 = t_2 = \dots = t_{j-1} = 0] \Rightarrow t_j > 0\} \cup \{0\}.$$

Let $\mathscr{M}(\mathbb{H}^{[n]})$ denote the Banach algebra of all complex Borel measures with support contained in $\mathbb{H}^{[n]}$, with the usual addition and scalar multiplication, and with convolution $*$, and the norm being the total variation of μ. It is shown that the maximal ideal space $X(\mathscr{M}(\mathbb{H}^{[n]}))$ of $\mathscr{M}(\mathbb{H}^{[n]})$, equipped with the Gelfand topology, is contractible as a topological space. In particular, it follows that $\mathscr{M}(\mathbb{H}^{[n]})$ is a projective free ring. In fact, for all subalgebras R of $\mathscr{M}(\mathbb{H}^{[n]})$ that satisfy a certain condition, it is shown that the maximal ideal space $X(R)$ of R is contractible. Several examples of such subalgebras are also given. We also show that this condition, although sufficient, is not necessary for the contractibility of unital subalgebras of $\mathscr{M}(\mathbb{H}^{[n]})$.

Mathematics subject classification (2000): Primary 54C40; Secondary 46J10, 32A38, 93D15.
Keywords and phrases: Contractibility of the maximal ideal space, convolution algebra of measures, projective-free ring, coprime factorization.

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Paper OaM-04-04

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