A MULTI–POINT DEGENERATE INTERPOLATION
PROBLEM FOR GENERALIZED SCHUR FUNCTIONS

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Abstract. The Nevanlinna-Pick-Carathéodory-Fejér interpolation problem with finitely many interpolation conditions is considered in the class $\mathscr{S}_\kappa$ of meromorphic functions f with κ poles inside the unit disk $\mathbb{D}$ and with $\|f\|_{L^\infty(\mathbb{T})} \leqslant 1$. Necessary and sufficient conditions for the existence and for the uniqueness of a solution are given in terms of the Pick matrix P of the problem explicitly determined from interpolation data. In particular it is shown that the problem admits infinitely many solutions if and only if κ is not less than the number of nonpositive eigenvalues of P. For κ equal to the number of nonpositive eigenvalues of P, we describe the solution set of the problem. Also we present necessary and sufficient conditions for the existence of a meromorphic function with a given pole multiplicity satisfying interpolation conditions and having the minimal possible L^∞-norm on the unit circle $\mathbb{T}$.

REFERENCES

[1] V. M. ADAMJAN, D. Z. AROV AND M. G. KREIN, *Analytic properties of the Schmidt pairs of a Hankel operator and the generalized Schur-Takagi problem*, Mat. Sb., **86** (1971), 34–75.

[2] D. ALPAY, T. CONSTANTINESCU, A. DIJKSMA AND J. ROVNYAK, *Notes on interpolation in the generalized Schur class. I. Applications of realization theory*, in: *Interpolation theory, systems theory and related topics*, Operator Theory Adv. Appl., **OT 134**, 67–97, Birkhäuser, Basel, 2002.

[3] J. A. BALL, *A non-Euclidean Lax-Beurling theorem with applications to matricial Nevanlinna-Pick interpolation*, in: *Toeplitz centennial*, Operator Theory: Adv. Appl., **OT 4**, 67–84, Birkhüser, Basel 1982.

[4] J. A. BALL, I. GOHBERG, AND L. RODMAN, *Interpolation of rational matrix functions*, Operator Theory: Adv. Appl., **OT 45**, Birkhäuser, Basel 1990.

[5] J. A. BALL, J. W. HELTON, *A Beurling-Lax theorem for the Lie group* U(m, n) *which contains most classical interpolation theory*, J. Operator Theory, **9**, 1 (1983), 107–142.

[6] V. BOLOTNIKOV, *On Carathéodory–Fejér problem for generalized Schur functions*, Integral Equations Operator Theory, **50**, 1 (2004), 9–41.

[7] V. BOLOTNIKOV, *On inertia of certain structured Hermitian matrices*, to appear in: *Numerical Methods for Structured Matrices and Applications. The Georg Heinig memorial volume* (D.A. Bini, V. Mehrmann, V. Olshevsky, E. Tyrtsyhnikov, M. van Barel, Eds.), Operator Theory: Adv. Appl., **OT 199**, Birkhäuser, Basel 2009.

[8] V. BOLOTNIKOV, *On a multi-point interpolation problem for generalized Schur functions*, Operator Theory: Adv. Appl., **OT 195**, 81–101, Birkhäuser, Basel 2010.

[9] V. BOLOTNIKOV, *Nevanlinna–Pick meromorphic interpolation: the degenerate case and minimal norm solutions*, J. Math. Anal. Appl., **353** (2009), 642–651.

[10] V. BOLOTNIKOV AND A. KHEIFETS, *On negative inertia of Pick matrices associated with generalized Schur functions*, Integral Equations Operator Theory, **56**, 3 (2006), 323–355.

[11] V. BOLOTNIKOV, A. KHEIFETS, AND L. RODMAN, *Jet functions having indefinite Carathéodory–Pick matrices*, Linear Algebra Appl., **385** (2004), 215–286.

[12] L. B. GOLINSKII, *A generalization of the matrix Nevanlinna–Pick problem*, Izv. Akad. Nauk Armyan. SSR Ser.Mat., **18** (1983), 187–205. (Russian).

[13] I.S. IOHVIDOV, *Hankel and Toeplitz matrices and forms. Algebraic theory*, Birkhäuser, Boston, Mass., 1982.

[14] M. KALTENBÄCK AND H. WORACEK, *The Krein formula for generalized resolvents in degenerated inner product spaces*, Monatsh. Math., **127**, 2 (1999), 119–140.

[15] M. G. KREĬN AND H. LANGER, *Über die verallgemeinerten Resolventen und die charakteristische Funktion eines isometrischen Operators im Raume Π_κ*, Colloq. Math. Soc. János Bolyai, **5** (1972), 353–399.

[16] M. G. KREĬN AND H. LANGER, *Über einige Fortsetzungsprobleme, die eng mit der Theorie hermitescher Operatoren im Raume Π_K zusammenhängen. I. Einige Funktionenklassen und ihre Darstellungen*, Math. Nachr., **77** (1977), 187–236.

[17] N. NIKOLSKII, *Treatise on the shift operator. Spectral function theory*, Grundlehren der Mathematischen Wissenschaften **273**, Springer–Verlag, Berlin, 1986.

[18] A. A. NUDELMAN, *A new problem of the type of the moment problem*, Dokl. Akad. Nauk SSSR, **233**, 5 (1977), 792–795.

[19] T. TAKAGI, *On an algebraic problem related to an analytic theorem of Carathéodory and Fejér*, Japan J. of Math., **1** (1924), 83–93.

[20] H. WORACEK, *Nevanlinna-Pick interpolation: the degenerated case*, Linear Algebra Appl., **252** (1997), 141–158.