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DERIVATIONS WHICH ARE INNER AS COMPLETELY BOUNDED MAPS

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Abstract. We consider derivations in the image of the canonical contraction θ_A from the Haagerup tensor product of a C^*-algebra A with itself to the space of completely bounded maps on A. We show that such derivations are necessarily inner if A is prime or if A is central. We also provide an example of a C^*-algebra which has an outer derivation implemented by an elementary operator.

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REFERENCES

[1] C. A. Akemann, G. A. Elliot, G. K. Pedersen, and J. Tomiyama, *Derivations and multipliers of C^*-algebras*, Amer. J. Math., **98** (1976), 679–708.
[2] S. D. Allen, A. M. Sinclair and R. R. Smith, *The ideal structure of the Haagerup tensor product of C^*-algebras*, J. reine angew. Math., **442** (1993), 111–148.
[3] P. Ara and M. Mathieu, *Local Multipliers of C^*-algebras*, Springer, London, 2003.
[4] R. J. Archbold, *Density theorems for the centre of a C^*-algebra*, J. London Math. Soc., **2**, 10 (1975), 189–197.
[5] R. J. Archbold, *Topologies for Primal Ideals*, J. London Math. Soc., **2**, 36 (1987), 524–542.
[6] R. J. Archbold and D. W. B. Somerset, *Quasi-standard C^*-algebras*, Math. Proc. Cambridge Philos. Soc, **107** (1990), 349–360.
[7] R. J. Archbold, D. W. B. Somerset and R. M. Timoney, *Completely bounded mappings and simplicial complex structure in the primitive ideal space of a C^*-algebra*, Trans. Amer. Math. Soc., **361** (2009), 1397–1427.
[8] R. J. Archbold, D. W. B. Somerset and R. M. Timoney, *On the central Haagerup tensor product and completely bounded mappings of a C^*-algebra*, J. Funct. Anal., **226** (2005), 406–428.
[9] B. Blackadar, *Operator Algebras*, Encyclopaedia of Mathematical Sciences 122, Springer-Verlag, Berlin, 2006.
[10] D. P. Blecher and C. Le Merdy, *Operator algebras and Their modules*, Clarendon Press, Oxford, 2004.
[11] C. Delaroche, *Sur les centres des C^*-algèbres*, Bull. Sc. Math., **91** (1967), 105–112.
[12] C. Delaroche, *Sur les centres des C^*-algèbres II*, Bull. Sc. Math., **92** (1968), 111–128.
[13] J. Dixmier, *C^*-algebras*, North-Holland, Amsterdam, 1977.
[14] J. M. G. Fell, *The structure of algebras of operator fields*, Acta Math., **106** (1961), 233–280.
[15] I. Kaplansky, *Normed algebras*, Duke Math. J., **16** (1949), 399–418.
[16] B. Magajna, *Uniform approximation by elementary operators*, Proc. Edinburgh Math. Soc., **52** (2009), 731–749.
[17] V. Paulsen, *Completely Bounded Maps and Operator Algebras*, Cambridge University Press, 2003.
[18] G. K. Pedersen, *C^*-algebras and their automorphism groups*, Academic Press, London, 1979.
[19] I. Raeburn and D. P. Williams, *Morita Equivalence and Continuous-Trace C^*-Algebras*, Mathematical Surveys and Monographs 60, Amer. Math. Soc., Providence, RI, 1998.
[20] S. Sakai, *C^*-algebras and W^*-algebras*, Springer-Verlag, 1971.
[21] R. R. Smith, *Completely Bounded Maps and the Haagerup Tensor Product*, J. Functional Analysis, **102** (1991), 156–175.

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[22] D. W. SOMERSET, *The central Haagerup tensor product of a C^*-algebra*, J. Operator Theory, **39** (1998), 113–121.

[23] D. W. SOMERSET, *The inner derivations and the primitive ideal space of a C^*-algebra*, J. Operator Theory, **29** (1993), 307–321.