

A NOTE ON k -PARANORMAL OPERATORS

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Abstract. It is still unknown whether the inverse of an invertible k -paranormal operator is normaloid, and so whether a k -paranormal operator is totally hereditarily normaloid. We provide sufficient conditions for the inverse of an invertible k -paranormal operator to be k -paranormal.

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REFERENCES

- [1] H.K. CHA, K.I. SHIN AND J.K. KIM, *On the superclasses of quasihyponormal operators*, J. Korea Soc. Math. Educ., **7** (2000), 79–86.
- [2] R.G. DOUGLAS, *Canonical models*, Topics in Operator Theory (Mathematical Surveys No.13, Amer. Math. Soc., Providence, 2nd pr. 1979), 161–218.
- [3] B.P. DUGGAL, *On dominant operators*, Arch. Math. (Basel), **46** (1986), 353–359.
- [4] B.P. DUGGAL, *On characterising contractions with C_{10} pure part*, Integral Equations Operator Theory, **27** (1997), 314–323.
- [5] B.P. DUGGAL AND S.V. DJORDJEVIĆ, *Generalized Weyl's theorem for a class of operators satisfying a norm condition*, Math. Proc. Royal Irish Acad., **104** (2004), 75–81.
- [6] B.P. DUGGAL, S.V. DJORDJEVIĆ AND C.S. KUBRUSLY, *Hereditarily normaloid contractions*, Acta Sci. Math. (Szeged), **71** (2005), 337–352.
- [7] B.P. DUGGAL AND C.S. KUBRUSLY, *Paranormal contractions have property PF*, Far East J. Math. Sci., **14** (2004), 237–249.
- [8] T. FURUTA, *Invitation to Linear Operators*, Taylor and Francis, London, 2001.
- [9] T. FURUTA, M. ITO AND T. YAMAZAKI, *A subclass of paranormal operators including class of log-hyponormal and several related classes*, Sci. Math., **1** (1998), 389–403.
- [10] V.I. ISTRĂȚESCU, *Introduction to Linear Operator Theory*, Marcel Dekker, New York, 1981.
- [11] M. ITO AND T. YAMAZAKI, *Relations between two inequalities $(B^{\frac{r}{2}} A^p B^{\frac{r}{2}})^{\frac{r}{p+r}} \geq B^r$ and $A^p \geq (A^{\frac{p}{2}} B^r A^{\frac{p}{2}})^{\frac{p}{p+r}}$ and their applications*, Integr. Equ. Operator Theory, **44** (2002), 442–450.
- [12] P.R. HALMOS, *Normal dilations and extensions of operators*, Summa Brasil. Math., **2** (1950), 125–134.
- [13] C.S. KUBRUSLY, *An Introduction to Models and Decompositions in Operator Theory*, Birkhäuser, Boston, 1997.
- [14] C.S. KUBRUSLY, *Hilbert Space Operators*, Birkhäuser, Boston, 2003.
- [15] C.S. KUBRUSLY AND B.P. DUGGAL, *On posinormal operators*, Adv. Math. Sci. Appl., **17** (2007), 131–147.
- [16] C.S. KUBRUSLY AND P.C.M. VIEIRA, *Strong stability for cohyponormal operators*, J. Operator Theory, **31** (1994), 123–127.
- [17] K. OKUBO, *The unitary part of paranormal operators*, Hokkaido Math. J., **6** (1977), 273–275.
- [18] C.R. PUTNAM, *Hyponormal contractions and strong power convergence*, Pacific J. Math., **57** (1975), 531–538.
- [19] H.C. RHALY, JR., *Posinormal operators*, J. Math. Soc. Japan, **46** (1994), 587–605.
- [20] A.L. SHIELDS, *Weighted shifts operators and analytic function theory*, Topics in Operator Theory (Mathematical Surveys No.13, Amer. Math. Soc., Providence, 2nd pr. 1979), 49–128b.

- [21] J.G. STAMPFLI AND B.L. WADHWA, *An asymmetric Putnam-Fuglede theorem for dominant operators*, Indiana Univ. Math. J., **25** (1976), 359–365.
- [22] J.G. STAMPFLI AND B.L. WADHWA, *On dominant operators*, Monatsh. Math., **84** (1977), 143–152.
- [23] B. SZ.-NAGY AND C. FOIAŞ, *Harmonic Analysis of Operators on Hilbert Space*, North-Holland, Amsterdam, 1970.
- [24] T. YOSHINO, *On the unitary part of dominant contractions*, Proc. Japan Acad. Ser. A Math. Sci., **66** (1990), 272–273.
- [25] T. YOSHINO, *Introduction to Operator Theory*, Longman, Harlow, 1993.